

# $d + id$ Holographic Superconductors

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## Abstract

A holographic model of  $d + id$  superconductors based on the action proposed by Benini, Herzog, and Yarom [arXiv:1006.0731] is studied. This model has a charged spin two field in an AdS black hole spacetime. Working in the probe limit, the normalizable solution of the spin two field in the bulk gives rise to a  $d + id$  superconducting order parameter at the boundary of the AdS. We calculate the fermion spectral function in this superconducting background and confirm the existence of fermi arcs for non-vanishing Majorana couplings. By changing the relative strength  $\gamma$  of the  $d$  and  $id$  condensations, the position and the size of the fermi arcs are changed. When  $\gamma = 1$ , the spectrum becomes isotropic and the spectral function is s-wave like. By changing the fermion mass, the fermi momentum is changed. We also calculate the conductivity for these holographic  $d + id$  superconductors where time reversal symmetry has been broken spontaneously. A non-vanishing Hall conductivity is obtained even without an external magnetic field.

## I. INTRODUCTION

The mechanism of the high temperature superconductors (SC's) is an unsolved mystery in physics [1]. High temperature SC's are layered compounds with copper-oxygen planes and are doped Mott insulators with strong electronic correlations. The pairing symmetry is unconventional and there is a strong experimental evidence showing that it is largely d-wave [2]. It is speculated that the pairing between electrons is mediated via strong anti-ferromagnetic spin fluctuations in the system. But the problem is difficult due to the strong-coupling nature of the system. Although significant progress has been made in the last many years [3, 4], alternative approaches may be valuable to tackle the problem.

An interesting alternative approach is the holographic correspondence between a gravitational theory and a quantum field theory, which first emerged under the framework of AdS/CFT correspondence [5–7]. This method has provided us a useful and complimentary framework to describe strong interacting systems without the sign problem (see e.g. [8–15]). In the original top-down approach within the AdS/CFT framework, both the gravity side and the field theory side were precisely known in string theory and gives us much deeper insight on this correspondence. But later in bottom-up approach we assume that the correspondence exists among the different pair of theories and try to make predictions from one side of the correspondence.

Recently, a gravitational model of hairy black holes [16, 17] has been used to model s-wave high temperature SC's [18–21]. In those class of models the Abelian symmetry of a complex scalar field is spontaneously broken (i.e. the Higgs mechanism) below some critical temperature. The Meissner effect was soon observed by including a magnetic field in the background [22, 23]. A similar construction of holographic multiband superconductor has also been considered recently in [24]. The multiband is coming from the condensation of a fundamental scalar field multiplet under non-abelian gauge group. The effect of the superconducting condensate on the holographic fermi surface has further been studied by calculating fermionic spectral functions [25–27]. Interestingly, the properties of spectral function appeared to have similar behavior to that found in the angle resolved photo-emission experiment. Analogously, holographic p-wave superconductors have also been proposed by coupling a  $SU(2)$  Yang-Mills field to the black hole background, where a vec-

tor hair develops in the superconducting phase [28–31]. Properties of fermionic spectral function has also been studied in p-wave superconducting background [32]. Also, there are attempts to build holographic d-wave SC's by spontaneously breaking the Abelian symmetry of a charged spin two field [33–36] and study different properties of the systems [37–39]. In addition to the bottom up approach mentioned above, there are various top-down constructions of those condensed matter like systems by considering D-brane configurations in the AdS black hole background in the string theory framework [40].

In this work we will be interested in studying more detailed properties of holographic d-wave superconductors using the action of [35, 36]. The most challenging problem to construct the holographic model of d-wave SC's is that the consistent action for the charged, massive spin two field in a general curve background is still not known. Although the action used in [35, 36] has some interesting features, such as having the right degrees of freedom and being ghost-free, it could be unstable or acausal for general gauge field configurations. It is argued that these problems may be cured by adding higher dimensional operators [36].

In this work, we will study properties of the holographic  $d+id$  SC's based on the action of [35, 36]. This is motivated by the observations of spontaneous breaking of time reversal invariance in, for example, the YBCO high temperature superconductor [41–45]. The time reversal breaking is thought to occur due to a complex combination of the d-wave condensates:  $d_{x^2-y^2} + id_{xy}$ . One of the possible interesting consequences of this complex d-wave condensation is that the system has a Hall conductivity even in the absence of an external magnetic field [46, 47]. By using the holographic framework, in this note we are interested in calculating the fermion spectral function, and the normal and Hall conductivity in the holographic  $d + id$  SC's.

Our paper is organized as follow: In section II we will first introduce our model following [36]. We consider the effective action for the charged spin two field in a four dimensional AdS-Schwarzschild black hole. In order for the system to be ghost free, we will consider the probe limit. Subsequently in section III we will obtain the normalizable background spin two field configuration which leads to a dual  $d + id$  superconducting phase of a strongly coupled system living on the boundary of AdS. Once we identify our required dual complex d-wave condensate background, we can study various linear transport properties of the system by considering linear order fermionic and gauge field perturbations. In section IV we will study the fermionic spectral function and discuss various properties depending

upon various parameters such as the Majorana coupling, mass of the fermion etc. In section VI we will study the conductivity of the superconducting background. As we have mentioned earlier, because of spontaneous violation of time reversal symmetry due to the background condensation, we found a non-vanishing optical Hall conductivity. In the final section we will conclude with some remarks and discuss future directions.

## II. THE MODEL

As we have mentioned in the introduction, the model that we are going to present is in the similar spirit of the holographic dual construction of an s-wave superconductor in an AdS black hole spacetime. It is well known that the phases of d-wave superconductivity can be described by a low energy effective theory of charged spin two tensor field in the framework of Landau-Ginzburg theory. At low temperature, the background solution of this charged spin two field yields the d-wave order parameter for the high temperature superconductivity. Analogously one particular construction of a holographic model of d-wave superconductor has been considered first in [33, 34] by taking a charged symmetric traceless spin-2 field in an AdS-black hole background. Subsequently, in spite of having long-standing technical and also conceptual problems, a more refined form of the charge spin-2 tensor field effective action has been proposed and studied in the context of pure d-wave superconductor [36]. Our main goal in this report is to study the properties of the more general  $d + id$  superconducting background in AdS/CFT framework. One of the theoretical motivations to consider these kinds of systems is to discover some universal low energy properties of general strongly coupled field theories. Keeping this motivation in mind, we start by considering the action in the bulk containing the gravity part and the matter part as

$$S = \frac{1}{2\kappa^2} \int d^4x \sqrt{-g} \left\{ \left( R + \frac{6}{L^2} \right) + \mathcal{L}_m \right\}, \quad (1)$$

where  $R$  is the Ricci scalar, the  $6/L^2$  term gives a negative cosmological constant and  $L$  is the AdS radius which will be set to unity in the units that we use.  $\kappa^2 = 8\pi G_N$  is the gravitational coupling. The Lagrangian for the matter fields includes a spin two field and a spin half fermion field. Both of them are charged under U(1) gauge field and can be massive. We write

$$\mathcal{L}_m = \mathcal{L}_b + \mathcal{L}_f. \quad (2)$$

The consistent construction of a charged spin two field theory in curved spacetime background is a long-standing problem. There have been lot of studies in constructing consistent interacting higher spin field theories [48]. As we have mentioned, the authors in Ref.[36] have proposed a unique Lagrangian for a charged spin-2 field in AdS space with the motivation in studying its dual field theoretic properties. In this report we will adopt their construction with the Lagrangian

$$\begin{aligned}\mathcal{L}_b = & -|D_\rho\varphi_{\mu\nu}|^2 + 2|D_\mu\varphi^{\mu\nu}|^2 + |D_\mu\varphi|^2 - [D_\mu\varphi^{*\mu\nu}D_\nu\varphi + \text{c.c.}] \\ & - m^2(|\varphi_{\mu\nu}|^2 - |\varphi|^2) + 2R_{\mu\nu\rho\lambda}\varphi^{*\mu\rho}\varphi^{\nu\lambda} - R_{\mu\nu}\varphi^{*\mu\lambda}\varphi_\lambda^\nu \\ & - \frac{1}{d+1}R|\varphi|^2 - iqF_{\mu\nu}\varphi^{*\mu\lambda}\varphi_\lambda^\nu - \frac{1}{4}F_{\mu\nu}F^{\mu\nu},\end{aligned}\tag{3}$$

where  $\varphi_{\mu\nu}$  is the spin two field with  $\varphi_{\mu\nu} = \varphi_{\nu\mu}$ ,  $\varphi = \varphi^\mu{}_\mu$ ,  $D_\mu$  is the covariant derivative ( $D_\alpha\varphi_{\mu\nu} = (\partial_\alpha + iqA_\alpha)\varphi_{\mu\nu}$  in flat space),  $d(=3)$  is the spatial dimension of the bulk and  $F_{\mu\nu}$  is the field strength tensor of the gauge field. As it has been discussed in detail in [36], the above action describes dynamics of the correct number of degrees of freedom only when the background is Einstein's manifold. So, the energy density of the higher spin field configuration should be very small compared to the background energy density. This essentially means we have to maintain the probe limit of spin-2 and fermion fields in a specific gravitation background. However, there still exists serious issues in dealing with the above action. For generic gauge field background, even though it has correct number of dynamical degrees of freedom, the equation motion loses its hyperbolicity or causality. The general belief is that these violations can be ameliorated by considering higher derivative operators in our Lagrangian. This requirement makes it very complicated to construct a fully consistent Lagrangian for the higher spin field in a curved background. However as has been argued in [36], we will adopt the effective field theory point of view, where all these effects could be very small in the limit of very low gauge field strength. This is essentially the limit in which we will be considering in our subsequent discussions.

In addition to above Lagrangian for the spin-2 field, we also consider the spin 1/2 fermion Lagrangian as

$$\mathcal{L}_f = i\bar{\Psi}(\Gamma^\mu D_\mu - m_\zeta)\Psi + \eta^*\varphi_{\mu\nu}^*\bar{\Psi}^c\Gamma^\mu D^\nu\Psi - \eta\bar{\Psi}\Gamma^\mu D^\nu(\varphi_{\mu\nu}\Psi^c).\tag{4}$$

The bulk Gamma matrices  $\Gamma^\mu$  satisfies the Clifford algebra  $\{\Gamma^\mu, \Gamma^\nu\} = 2g^{\mu\nu}$ . The  $U(1)$  gauge symmetry demands that the charges of the fermion and spin two field are related

by  $2q_\zeta = q$ . It is known that  $\eta$  dependent Majorana coupling helps the fermion spectral function to develop a gap. It has been argued that although there are more terms of the same dimension as these  $\eta$  terms, only  $\eta$  dependent terms give rise to an anisotropic gap. Thus, we have dropped the other terms for simplicity.

Under the field redefinition

$$\begin{aligned} A_\mu &\rightarrow A_\mu/q, \quad \varphi_{\mu\nu} \rightarrow \varphi_{\mu\nu}/q, \\ \Psi &\rightarrow \Psi/q, \quad \eta \rightarrow q\eta, \end{aligned} \tag{5}$$

$\mathcal{L}_m$  can then be written as

$$\mathcal{L}_m \rightarrow \mathcal{L}_m/q^2. \tag{6}$$

Thus, as  $q \rightarrow \infty$  while the  $\mathcal{L}_m$  remains finite, we can work in the so called probe limit to ignore the back reaction of the matter fields  $A_\mu$ ,  $\varphi_{\mu\nu}$ , and  $\Psi$ . Also, in the probe limit, the  $q$  dependence of observables can be recovered through the above simple scaling. Thus, we will work in the probe limit from now on. In this limit, the gravitational field equation satisfies the Einstein equation

$$R_{\mu\nu} = \frac{2\Lambda}{d-1} g_{\mu\nu}, \tag{7}$$

where  $\Lambda = -3/L^2$ . This yields the AdS Schwarzschild black hole solution with the metric

$$ds^2 = \frac{L^2}{z^2} \left( -f(z)dt^2 + d\vec{x}_{d-1}^2 + \frac{dz^2}{f(z)} \right), \tag{8}$$

with

$$f(z) = 1 - \left( \frac{z}{z_h} \right)^d, \tag{9}$$

where  $z = z_h$  is the black hole horizon and  $z = 0$  is the boundary. The Hawking temperature can be expressed as

$$T = \frac{d}{4\pi z_h}. \tag{10}$$

An important point to note is that, the action is invariant under the scaling transformations

$$\begin{aligned} (t, x, y, z, T) &\rightarrow (ct, cx, cy, cz, T/c), \\ (A_\mu, \varphi_{\mu\nu}, \Gamma^\mu, \Psi) &\rightarrow (A_\mu/c, \varphi_{\mu\nu}/c^2, c\Gamma^\mu, \Psi), \end{aligned} \tag{11}$$

which determine the conformal dimension of each field. In the following section, we will consider this black hole background in order to solve the equations of motion for the spin-2 and electromagnetic gauge field in the probe limit.

### III. $d + id$ CONDENSATE

As we know from the standard AdS/CFT dictionary, the bulk massive spin-2 field corresponds to a spin-2 operator in the boundary field theory with fixed conformal dimension. The conformal dimension is fixed by the spin-2 mass. This boundary spin-2 dual operator under boundary Lorentz transformations is to be identified with the d-wave order parameter. Our main interest here is to describe the d-wave SC in the dual boundary field theory. Now since the spacetime background that we have considered has a translation symmetry in the boundary direction, the condensation on the  $x$ - $y$  plane on the boundary should also have translational invariance. However rotational symmetry should be spontaneously broken down to  $Z(2)$  with the d-wave like condensate changing its sign under a  $\pi/2$  rotation on the  $x$ - $y$  plane. To incorporate these features, we use an ansatz for the symmetric traceless  $\varphi_{\mu\nu}$ :

$$\begin{aligned}\varphi_{xx} &= -\varphi_{yy} = \frac{1}{2z^2} \psi_1(z), \\ \varphi_{xy} &= \varphi_{yx} = \frac{1}{2z^2} \psi_2(z),\end{aligned}\tag{12}$$

and  $\varphi_{\mu\nu} = 0$  for  $\mu, \nu \neq x, y$ , and for the gauge field  $A_\mu$ :

$$A = A_\mu dx^\mu \equiv \phi(z) dt.\tag{13}$$

Under a  $\theta$  angle rotation in the  $x$ - $y$  plan,

$$\begin{pmatrix} \varphi_{xx} \\ \varphi_{xy} \end{pmatrix} \rightarrow \begin{pmatrix} \cos 2\theta & -\sin 2\theta \\ \sin 2\theta & \cos 2\theta \end{pmatrix} \begin{pmatrix} \varphi_{xx} \\ \varphi_{xy} \end{pmatrix}.\tag{14}$$

If  $\varphi_{xy}/\varphi_{xx}$  is real, then we can always rotate the coordinates in the  $x$ - $y$  plan such that  $\varphi_{xy} = 0$  and then make  $\varphi_{xx}$  real by a gauge transformation. Otherwise  $\varphi_{xx}$  and  $\varphi_{xy}$  will both exist. The equations of motion of the gauge field and the spin two field are

$$\phi'' + \frac{3-d}{z} \phi' - \frac{q^2}{z^2 f} (|\psi_1|^2 + |\psi_2|^2) \phi = 0,\tag{15a}$$

$$\psi_i'' + \left(\frac{f'}{f} - \frac{d-1}{z}\right) \psi_i' + \left(\frac{q^2 \phi^2}{f^2} - \frac{m^2}{z^2 f}\right) \psi_i = 0, \quad i = 1, 2.\tag{15b}$$

The coupling to the fermion field through  $\delta \mathcal{L}_f / \delta \varphi_{\mu\nu}^*$  is dropped because there is no fermion condensate.

Near the horizon ( $\delta z = z - z_h \rightarrow 0$ ),  $f(z) \simeq -d\delta z/z_h$ , and

$$\begin{aligned}\phi'' + \frac{q^2}{3z_h\delta z} (|\psi_1|^2 + |\psi_2|^2) \phi &\simeq 0, \\ \psi_i'' + \frac{1}{\delta z} \psi_i' + \frac{z_h^2 q^2 \phi^2}{9\delta z^2} \psi_i &\simeq 0,\end{aligned}\tag{16a}$$

which yields

$$\begin{aligned}\phi &\simeq \frac{c_1}{z_h^2} \delta z, \\ \psi_i &\simeq c_{2,i} + c_{3,i} \ln \delta z.\end{aligned}\tag{17}$$

The finiteness of  $\psi_i$  at the horizon sets  $c_{3,i} = 0$ .

Near the boundary  $z \rightarrow 0$ ,  $f \rightarrow 0$ , and

$$\begin{aligned}\phi'' - \frac{q^2}{z^2} (|\psi_1|^2 + |\psi_2|^2) \phi &\simeq 0, \\ \psi_i'' - \frac{2}{z} \psi_i' - \frac{m^2}{z^2} \psi_i &\simeq 0.\end{aligned}\tag{18a}$$

$\psi_i$  has the asymptotic solution with two terms:

$$\psi_i \simeq c_{4,i} z^{\Delta_+} + c_{5,i} z^{\Delta_-}, \quad \Delta_{\pm} = \frac{3 \pm \sqrt{9 + 4m^2}}{2}.\tag{19}$$

One of the terms can be identified as the source and the other term identified as the corresponding condensate. In other words, the asymptotic behavior of the spin two field near the boundary is

$$\psi_i(z) = z^{d-\Delta} \left[ \psi_i^{(s)} + O(z) \right] + z^{\Delta} \left[ \frac{\langle \mathcal{O}_i \rangle}{2\Delta - d} + O(z) \right]\tag{20}$$

where  $\psi_i^{(s)}$  is the source,  $\langle \mathcal{O}_i \rangle$  is the condensate, and  $\mathcal{O}_i$  is the field theory operator that  $\psi_i$  couples to at the boundary.  $m^2 = \Delta(\Delta - d)$  and  $\Delta$  is conformal dimension of  $\langle \mathcal{O}_i \rangle$  (because the conformal dimension for  $\psi_i$  is zero).

Depending on the size of  $m^2$ , there could be different scenarios:

(a) If  $m^2 > 0$ ,  $\Delta_- < 0$ , we need to set  $c_{5,i} = 0$  (the sourceless condition) to keep  $\psi_i$  finite at the boundary. Thus,  $\phi$  has the asymptotic solution

$$\phi \simeq \mu + \rho z + O(z^2),\tag{21}$$

where the physical meaning of  $\mu$  is the chemical potential while  $\rho$  is the corresponding charge density. We can always use the scaling of Eq. (11) to set  $\mu = 1$ . In this case



$\psi_2(z)/\psi_1(z)$  is a  $z$  independent constant. Because the combination  $c_{4,2}\psi_1 - c_{4,1}\psi_2$  vanishes near the boundary. It continues to vanish for all  $z$  by the equations of motion.

(b) If  $0 > m^2 > -9/4$ ,  $\Delta_{\pm} > 0$ , one can either chose the  $z^{\Delta+}$  or  $z^{\Delta-}$  term to be the condensate. If we choose the  $z^{\Delta+}$  term to be the condensate for  $\psi_1$ , and choose the  $z^{\Delta-}$  term to be the condensate for  $\psi_2$ , then Eq.(21) is still true. But then  $\psi_2(z)/\psi_1(z)$  is no longer a constant.

(c) If  $m^2 = 0$ ,  $\Delta_- = 0$ , if we choose the  $z^{\Delta+}$  term to be the source, then Eq.(21) is no longer true. Hence, the physical meaning in this case is unclear. The other choice of the source goes back to case (a).

In case (b), the two condensates have different dimensions at the boundary. This might imply competition between the two order parameters in the system. However, the scaling dimension of any primary operator of a conformal field theory has a unitarity bound [49]. As has been argued in [36], this unitarity bound constrains mass of a spin-2 field as  $m^2 \geq 0$ . Thus, in this report, we will just focus on case (a), where

$$\begin{aligned}\psi_1(z) &= iQ_1\psi(z), \quad \psi_2(z) = Q_2\psi(z), \\ |Q_1|^2 + |Q_2|^2 &= 1, \quad \psi(z) = \psi^*(z).\end{aligned}\tag{22}$$

We have solved the equations of motion for the spin-2 field  $\psi$  and gauge field  $\phi$  using standard shooting algorithm by demanding  $\mu = 1$  and the normalizability of  $\psi$  near the asymptotic boundary. We found the critical temperature  $T_c = \frac{d}{4\pi z_{hc}}$  for a fixed chemical potential and charge, below which non-trivial bulk profile for the spin-2 field component exists. According to AdS/CFT correspondence, the d-wave condensation  $\langle \mathcal{O} \rangle$  in the dual boundary field theory is identified with the coefficient of the normalizable solution of  $\psi$ . In Fig.1 we show the condensate field ( $\langle \mathcal{O} \rangle = \sqrt{|\langle \mathcal{O}_1 \rangle|^2 + |\langle \mathcal{O}_2 \rangle|^2}$ ) disappears above the critical temperature  $T_c$ . Once we know the background condensation, it would be interesting to see the fermionic response function which essentially captures the properties of the background condensation. On the other hand, the electromagnetic perturbation is essential to study the conductivity of this background. In what follows we will first study the fermionic spectral function and discussed about the gap structure of the underlying strongly coupled system depending upon the type of fermionic coupling with a background we choose. Using the scaling relations in Eqs.(5) and (11), one can show that the combination  $\langle q\mathcal{O} \rangle^{1/\Delta} \mu/\rho$  is independent of  $q$  and  $\mu$ .

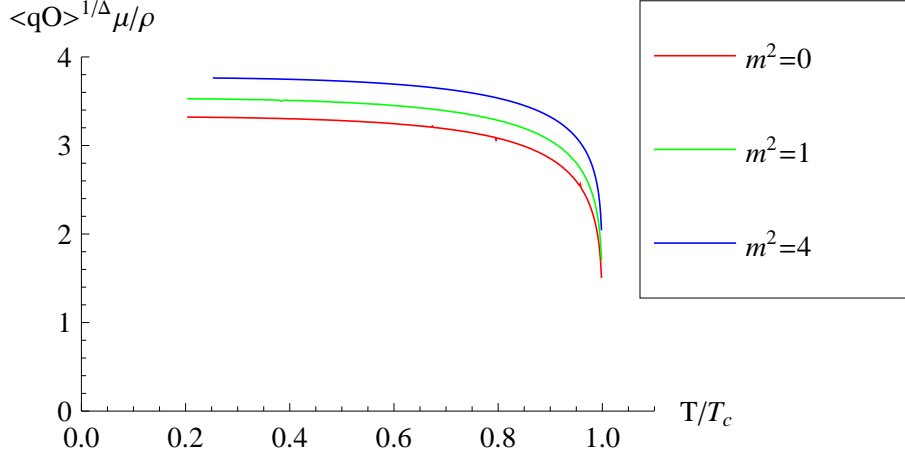


FIG. 1:  $\langle q\mathcal{O} \rangle^{1/\Delta} \mu/\rho$  vs.  $T/T_c$  is shown. The combination in the  $y$ -axis is independent of  $q$  and  $\mu$ .

#### IV. FERMION SPECTRAL FUNCTION

The equation of motion for the bulk fermion field looks like

$$0 = (\Gamma^\mu D_\mu - m_\zeta) \Psi + 2i\eta\varphi_{\mu\nu}\Gamma^\mu D^\nu \Psi^c + i\eta\varphi_\mu\Gamma^\mu \Psi^c, \quad (23)$$

where  $\varphi_\mu = D^\nu \varphi_{\nu\mu}$  and the covariant derivative on the spinor field is

$$D_\mu \Psi = \left( \partial_\mu + \frac{1}{4} \omega_{\mu, \underline{\lambda}\underline{\sigma}} \Gamma^{\underline{\lambda}\underline{\sigma}} - iq_\zeta A_\mu \right) \Psi \quad (24)$$

with  $\omega$  the spin connection,  $\Gamma^{\mu\nu} \equiv \Gamma^{[\underline{\mu}\underline{\nu}]}$ , and with the vielbein indices underlined. We have chosen the bulk Gamma matrices  $\Gamma^\mu$  representation

$$\begin{aligned} \Gamma^{\underline{t}} &= \begin{pmatrix} -i\sigma_2 & 0 \\ 0 & i\sigma_2 \end{pmatrix}, \Gamma^{\underline{z}} = \begin{pmatrix} \sigma_3 & 0 \\ 0 & \sigma_3 \end{pmatrix}, \\ \Gamma^{\underline{x}} &= \begin{pmatrix} \sigma_1 & 0 \\ 0 & \sigma_1 \end{pmatrix}, \Gamma^{\underline{y}} = \begin{pmatrix} 0 & -i\sigma_2 \\ i\sigma_2 & 0 \end{pmatrix} \end{aligned} \quad (25)$$

It is convenient to rescale  $\zeta = (-g \cdot g^{zz})^{1/4} \Psi$  which removes the spin connection completely, and to work in the momentum space with the Fourier components:

$$\zeta = e^{-i\omega t + i\vec{k} \cdot \vec{x}} \zeta^{(\omega, \vec{k})}(z) + e^{i\omega t - i\vec{k} \cdot \vec{x}} \zeta^{(-\omega, -\vec{k})}(z). \quad (26)$$

We then decompose the four-component spinor to two two-component spinors:  $\zeta = (\zeta_1, \zeta_2)$ , such that Eq. (23) can be rewritten as

$$\begin{aligned} D_{(1)}\zeta_1^{(\omega, k_x)} + 2\eta(g^{xx})^{3/2}k_x \left[ \varphi_{xx-yy}\sigma_1\zeta_1^{(-\omega, -k_x)*} - i\varphi_{xy}\sigma_2\zeta_2^{(-\omega, -k_x)*} \right] &= 0 \\ D_{(2)}\zeta_2^{(\omega, k_x)} + 2\eta(g^{xx})^{3/2}k_x \left[ \varphi_{xx-yy}\sigma_1\zeta_2^{(-\omega, -k_x)*} + i\varphi_{xy}\sigma_2\zeta_1^{(-\omega, -k_x)*} \right] &= 0 \end{aligned} \quad (27)$$

where

$$D_{(\alpha)} = \sqrt{g^{zz}}\sigma_3\partial_z - m_\zeta + (-1)^\alpha(\omega + q_\zeta A_t)\sqrt{-g^{tt}}\sigma_2 + ik_x\sqrt{g^{xx}}\sigma_1. \quad (28)$$

Without losing generality, we set  $k_y = 0$  while the condensate can have arbitrary orientations.

The spectral function is defined by the imaginary part of the trace of the retarded green's function

$$A(\omega, \mathbf{k}) \equiv \text{Im}[\text{Tr}(G_R)]. \quad (29)$$

To compute  $A(\omega, \mathbf{k})$ , we follow the method developed in [50–52] and applied in [26] which leads to the simple “flow equation”:

$$\begin{aligned} &\sqrt{g^{zz}}\partial_z(iG) \\ &= 2m_\zeta(iG) + \left[ -ik_x\sqrt{g^{xx}}\mathcal{A} + i(\omega\mathcal{B} + q_\zeta A_t\mathcal{C})\sqrt{-g^{tt}} + 2k_x\eta(g^{xx})^{3/2}\mathcal{D} \right] \\ &\quad - (iG) \left[ ik_x\sqrt{g^{xx}}\mathcal{A} + i(\omega\mathcal{B} + q_\zeta A_t\mathcal{C})\sqrt{-g^{tt}} + 2k_x\eta(g^{xx})^{3/2}\mathcal{E} \right] (iG), \end{aligned} \quad (30)$$

where

$$\begin{aligned} \mathcal{A} &= \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}, \mathcal{B} = \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}, \\ \mathcal{C} &= \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}, \mathcal{D} = \begin{pmatrix} 0 & 0 & \varphi_{xx} & -\varphi_{xy} \\ 0 & 0 & \varphi_{xy} & \varphi_{xx} \\ \varphi_{xx}^* & -\varphi_{xy}^* & 0 & 0 \\ \varphi_{xy}^* & \varphi_{xx}^* & 0 & 0 \end{pmatrix}, \end{aligned} \quad (31)$$

and  $\mathcal{E} = \mathcal{D}^\dagger$ . The initial condition which satisfies the ingoing wave boundary condition at the horizon of an AdS black hole would be

$$G_R = z^{2m_\zeta} G|_{z \rightarrow 0}. \quad (32)$$

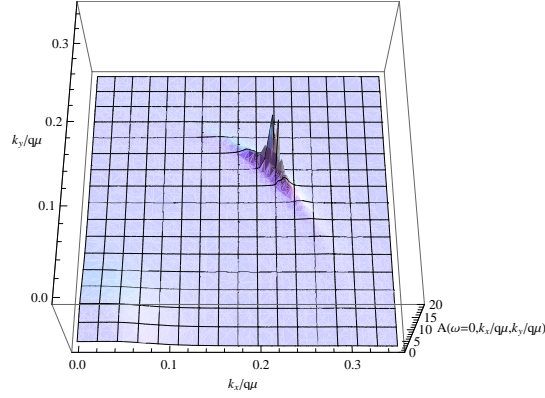


FIG. 2: The appearance of the fermi arcs in the fermion spectral density  $A(\omega = 0, k_x/q\mu, k_y/q\mu)$  for normal d-wave superconductor ( $\varphi_{xy} = 0$ ) with  $m = 0$ ,  $m_\zeta = 0$ ,  $\eta = 0.15q$  and  $T = 0.66T_c$ .

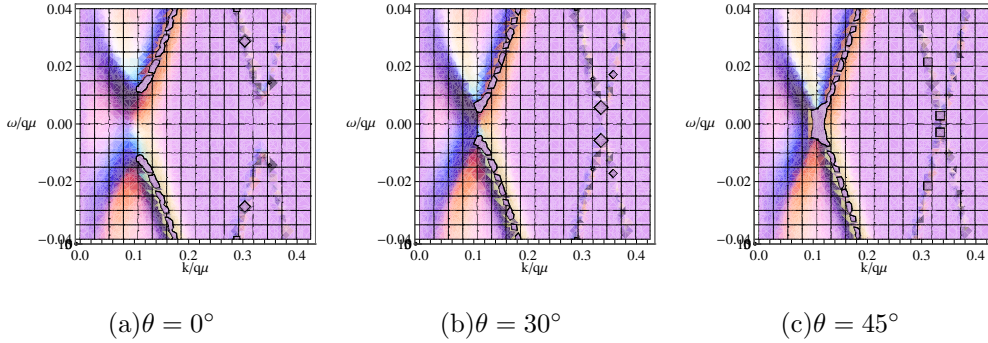


FIG. 3: The spectral function  $A(\omega/q\mu, k/q\mu)$  shown for various angles  $\theta$  with respect to the  $x$ -axis for  $m^2 = 2$ ,  $T = 0.66T_c$  and  $\eta = 0.15q$ .

Using the scaling relations in Eqs.(5) and (11), one can show that the spectral function  $A(\omega/q\mu, \mathbf{k}/q\mu)$  is independent of  $q$  and  $\mu$ , provided  $\eta/q$  is also  $q$  independent.

### A. Fermi Arc

As mentioned above, when  $\varphi_{xy}/\varphi_{xx}$  is real, we can always rotate the coordinate in the  $x$ - $y$  plan such that  $\varphi_{xy} = 0$  and make  $\varphi_{xx}$  real by a gauge transformation. The corresponding fermion spectral function  $A(\omega = 0, \mathbf{k}/q\mu)$  for  $\varphi_{xy} = 0$ ,  $\eta = 0.15q$ ,  $m = 0$ ,  $m_\zeta = 0$ , and  $T = 0.66T_c$  is shown in Fig. 2.

This case was first studied in Ref.[35]. It was found that the spectral function at  $\omega = 0$  does not vanish in the nodal direction but vanishes (up to some small values at small

$k = |\mathbf{k}|$ ) in other directions. This suggests that the system is gapped except in the nodal direction for  $T < T_1$ . At higher  $T$ , the ungapped directions (the so called “fermi arc”) increase and eventually the system becomes ungapped above  $T_2$  (with  $T_2 < T_c$ ). This is reflected in the growing angular size of fermi arc with non-zero spectral function at  $T_1 < T < T_2$ . In this region, a typical spectral function  $A(\omega/q\mu, k/q\mu)$  for various angles  $\theta$  with respect to the  $x$ -axis, is shown in Fig. 3. Up to the small structure at low  $k$ , one can see the gap-like structure at  $0^\circ$  and  $30^\circ$  and the vanishing of the gap at  $45^\circ$ . At  $0^\circ$ , the spectral width is significantly smaller than the gap, suggesting it is a fermi liquid. One can also identify the fermi momentum at  $k/q\mu \simeq 0.3$ . In this model, the angular dependence of the fermi momentum is not built in. It would be interesting to generalize this model to incorporate this feature in the future.

At this point we want to note the appearance of another inner fermi arc in the fermionic spectra function for a higher value of spin two field mass. This is also clearly visible in Fig. 3. As we decrease the mass of the background spin two field, this inner fermi surface disappears but outer fermi arc still exist. It is interesting to study this in more details and possibly ascribe any physical significance to this. Apparently the existence of two fermi surfaces suggest that in the boundary field theory, we may have two different characteristic fermionic degrees of freedom.

## B. Dirac Mass Dependence

While it is interesting to see the fermi arcs in this holographic model, however, experimentally, the fermi arcs in high temperature SC’s happen at  $T_1 = T_c < T < T_2$ , i.e. fermi arcs happen in the pseudo gap phase but not in the SC phase [54–56].

With the goal of eliminating the fermi arc in the SC phase in mind, we study the dirac mass ( $m_\zeta$ ) effect to the spectral function. We found that while  $m_\zeta$  tends to increase the gap, it also makes the nodal points not gapless any more.

In Fig.4, it is shown that the fermi surface shrinks and height of the peak is reduced when we turn on  $m_\zeta$ . If  $m_\zeta$  is sufficiently large, the spectral function will be gapped in all directions.

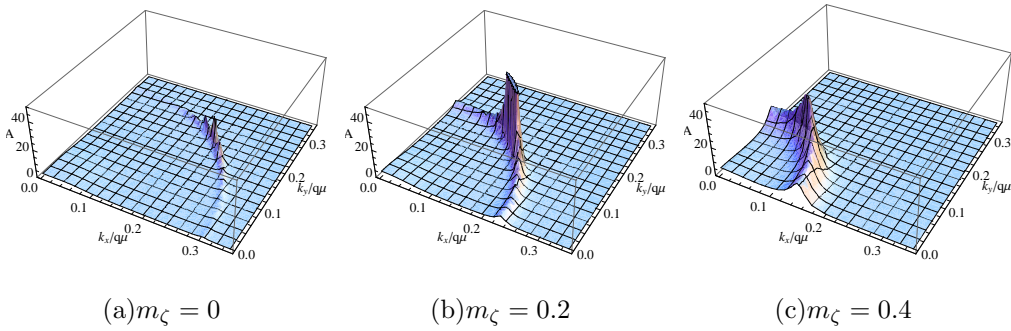


FIG. 4: The dirac mass dependence of the spectral function  $A(\omega = 0, \mathbf{k}/q\mu)$ . The fermi momentum decreases when  $m_\zeta$  is reduced.  $m^2 = 0, T = 0.66T_c$  and  $\eta = 0.05q$ .

## V. $d + id$ SUPERCONDUCTORS

As mentioned above, if  $\varphi_{xy}/\varphi_{xx}$  is not real, then we cannot rotate the coordinates such that  $\varphi_{xy} = 0$ , such that both  $\varphi_{xx}$  and  $\varphi_{xy}$  will exist. A particular interesting case is  $\varphi_{xy}/\varphi_{xx} = \pm i$ , whose ratio is unchanged under the rotation in Eq. (14). This means the difference between  $\varphi_{xx}$  and  $\varphi_{xy}$  in all directions is just a common phase which can be gauged away without physical consequences. This implies the fermion spectral function will be the same in all directions.

In Fig.5, we plot  $A(\omega = 0, \mathbf{k})$  for different

$$\gamma \equiv \frac{i\varphi_{xy}}{\varphi_{xx}}. \quad (33)$$

As expected, the  $\gamma = 0$  case is just a  $\pi/4$  rotation of the  $\gamma \rightarrow \infty$  case. Both of them are purely d-wave SC's. In the  $d + id$  SC with  $\gamma = 1$ , the spectral function looks like a s-wave SC.

## VI. CONDUCTIVITY

In this section we will calculate the conductivity by turning on an electromagnetic perturbation as previously discussed. According to the standard AdS/CFT dictionary, the gauge field perturbation in the bulk will lead to a boundary current. The boundary value of the perturbed gauge field becomes a source for this boundary current.

The conductivity tensor  $\sigma_{ij}$  can be defined as

$$J_i = \sigma_{ij} E_j, \quad (34)$$

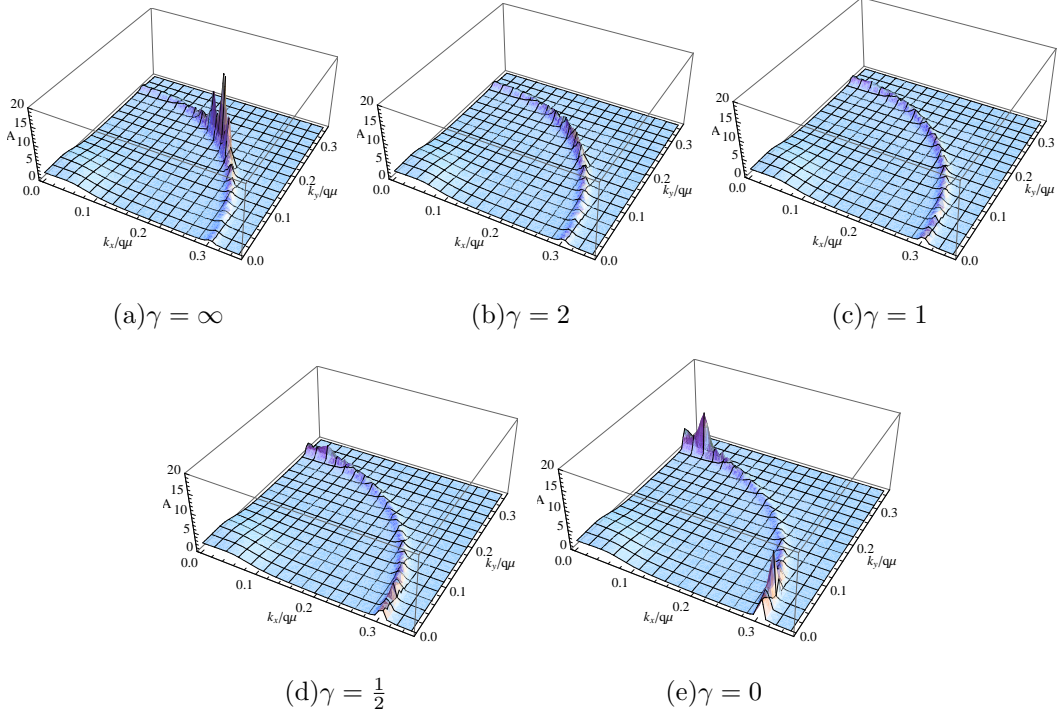


FIG. 5:  $A(\omega = 0, \mathbf{k}/q\mu)$  for  $d+id$  superconductors. For  $\gamma = \infty$  and  $\gamma = 0$ , the spectral functions reduce to the d-wave case. For,  $\gamma = 1$ , the spectral function is isotropic and is s-wave like.  $m^2 = 0, T = 0.66T_c, \eta = 0.05q, m_\zeta = 0$

where  $E_i$  and  $J_i$  are the external electric field and the induced current, respectively, in the  $i$ -direction ( $i = x, y$ ). In linear response theory,  $\sigma_{ij}$  is a current-current correlator which can be schematically denoted as  $\sigma_{ij} \sim \langle \Omega | [J_i, J_j] | \Omega \rangle$ , where the matrix element denotes an ensemble average. Under a  $\pi/2$  rotation along the  $z$ -axis ( $R$ ),  $R^{-1}J_iR = \epsilon_{ij}J_j$ , where  $\epsilon_{ij}$  is an anti-symmetric tensor, and assuming the ensemble average is governed by properties of the ground state which is in general a  $d+id$  condensation, we have  $R|\Omega\rangle = -|\Omega\rangle$ . Then,  $\sigma_{ij} \sim \langle \Omega | R R^{-1} [J_i, J_j] R R^{-1} | \Omega \rangle = \langle \Omega | R^{-1} [J_i, J_j] R | \Omega \rangle = \langle \Omega | [\epsilon_{ik}J_k, \epsilon_{jl}J_l] | \Omega \rangle$ . This implies  $\sigma_{xx} = \sigma_{yy}$  and  $\sigma_{xy} = -\sigma_{yx}$ .

Under a parity ( $P_x$ ) transformation with respect to the  $x$ -axis, the condensates transform as

$$P_x \begin{pmatrix} \langle \mathcal{O}_1 \rangle & \langle \mathcal{O}_2 \rangle \\ \langle \mathcal{O}_2 \rangle & -\langle \mathcal{O}_1 \rangle \end{pmatrix} \rightarrow \begin{pmatrix} \langle \mathcal{O}_1 \rangle & -\langle \mathcal{O}_2 \rangle \\ -\langle \mathcal{O}_2 \rangle & -\langle \mathcal{O}_1 \rangle \end{pmatrix}. \quad (35)$$

If  $\langle \mathcal{O}_2 \rangle = 0$ , as the case which is always achievable in a d-wave SC, then  $P_x|\Omega\rangle = \pm|\Omega\rangle$ . Thus,  $\sigma_{ij} \sim \langle \Omega | P_x P_x^{-1} [J_i, J_j] P_x P_x^{-1} | \Omega \rangle = \langle \Omega | P_x^{-1} [J_i, J_j] P_x | \Omega \rangle \sim (-1)^{\delta_{iy} + \delta_{jy}} \sigma_{ij}$ . This yields  $\sigma_{xy} = \sigma_{yx} = 0$ . In the case of  $d+id$ , however,  $|\Omega\rangle$  is not an eigenstate of  $P_x$ . Then,

in general  $\sigma_{xy} = -\sigma_{yx} \neq 0$ . A similar argument using time reversal symmetry yields the same conclusion. So unlike the normal holographic d-wave SC, in addition to standard  $\sigma_{xx}$ , the  $d + id$  SC could have a non-vanishing Hall conductivity  $\sigma_{xy}$  component.

In order to make our analysis simple, using the above symmetry we can diagonalize our conductivity tensor such that Eq.34 becomes

$$J_{\pm} = \sigma_{\mp} E_{\pm}, \quad (36)$$

where

$$J_{\pm} = J_x \pm iJ_y ; E_{\pm} = E_x \pm iE_y ; \sigma_{\pm} = \sigma_{xx} \pm i\sigma_{xy}. \quad (37)$$

As we have mentioned before, in order to calculate the conductivity, we need to consider the linearized perturbations in the bulk black hole background [53]. A consistent set of gauge field and spin-2 field perturbations are

$$\begin{aligned} \delta A(t, z) &= e^{-i\omega t} (a_x(z)dx + a_y(z)dy), \\ \delta\varphi_{\mu\nu}(t, z) &= e^{-i\omega t} \begin{pmatrix} 0 & 0 & \xi_{tx}(z) & \xi_{ty}(z) \\ 0 & 0 & \xi_{rx}(z) & \xi_{ry}(z) \\ \xi_{tx}(z) & \xi_{rx}(z) & 0 & 0 \\ \xi_{ty}(z) & \xi_{ry}(z) & 0 & 0 \end{pmatrix}, \end{aligned} \quad (38)$$

where  $\omega$  is the frequency of the perturbation. Similarly, we can define the perturbation for the complex conjugate field  $\delta\varphi_{\mu\nu}^*$  also. In the above perturbation ansatz we have only considered zero momentum modes. We only limit ourself to consider the case where  $\gamma = i\varphi_{xy}/\varphi_{xx} = Q_2/Q_1$  is real.

With this perturbation ansatz we would like to obtain the linear order perturbation equations. Apparently as one can observe from [36] that when the background has only d-wave condensation, the equation of motion for two independent gauge field perturbations  $a_x$  and  $a_y$  are not mixed each other. From this one can arrive at the conclusion that there is no Hall current in the dual superfluid phase. But in a  $d + id$  SC,  $a_x$  and  $a_y$  couple which leads to Hall conductivity. The equation of motion in this general background is complicated to solve. But two decoupled set of equations emerge if we perform the



following field redefinition:

$$\begin{aligned}
a_m &= a_x + ia_y, \quad a_p = a_x - ia_y, \\
\xi_{mt} &= \xi_{ty} - i\xi_{tx}, \quad \xi_{pt} = \xi_{ty} + i\xi_{tx}, \\
\xi_{mz} &= \xi_{zy} - i\xi_{zx}, \quad \xi_{pz} = \xi_{zy} + i\xi_{zx}, \\
\xi_{mt}^* &= \xi_{ty}^* - i\xi_{tx}^*, \quad \xi_{pt}^* = \xi_{ty}^* + i\xi_{tx}^*, \\
\xi_{mz}^* &= \xi_{zy}^* - i\xi_{zx}^*, \quad \xi_{pz}^* = \xi_{zy}^* + i\xi_{zx}^*.
\end{aligned} \tag{39}$$

Under this field redefinition one set of decoupled equations of motion become

$$\begin{aligned}
a_m'' + \frac{f'}{f}a_m' + \frac{\omega^2}{f^2}a_m + \frac{q\psi}{2f^2}[(Q_2 - Q_1)\xi_{mt}^* - (Q_2 + Q_1)\xi_{mt}] - i\frac{q\psi}{2}[(Q_2 - Q_1)\xi_{mz}^* \\
- (Q_2 + Q_1)\xi_{mz}] + i\frac{q}{2f}(\psi'f - f'\psi)[(Q_2 - Q_1)\xi_{mz}^* - (Q_2 + Q_1)\xi_{mz}] = 0, \\
\xi_{mt}'' + \frac{2}{z}\xi_{mt}' - \frac{2f + L^2m^2}{z^2f}\xi_{mt} + \frac{L^2q\psi}{4z^2f}(Q_2 + Q_1)(\omega + 2q\phi)a_m \\
+ \frac{i}{2}[(2(\omega + q\phi)\xi_{mz}' + q\phi'\xi_{mz})] = 0, \\
\xi_{mz}[z^2(\omega + q\phi)^2 - m^2L^2f] + i\frac{L^2qf}{4}(Q_1 + Q_2)(\psi a_m' + 2\psi'a_m) \\
- iz^2(\omega + q\phi)\xi_{mt}' - i\frac{z}{2}(4\omega + 4q\phi + qz\phi')\xi_{mt} = 0, \\
\xi_{mt}'' + \frac{2}{z}\xi_{mt}' - \frac{2f + L^2m^2}{z^2f}\xi_{mt} + \frac{L^2q\psi}{4z^2f}(Q_2 - Q_1)(-\omega + 2q\phi)a_m \\
- \frac{i}{2}[(2(-\omega + q\phi)\xi_{mz}^* + q\phi'\xi_{mz}^*)] = 0, \\
\xi_{mz}^*[z^2(-\omega + q\phi)^2 - m^2L^2f] - i\frac{L^2qf}{4}(Q_2 - Q_1)(\psi a_m' + 2\psi'a_m) \\
+ iz^2(-\omega + q\phi)\xi_{mt}' + i\frac{z}{2}(-4\omega + 4q\phi + qz\phi')\xi_{mt}^* = 0,
\end{aligned} \tag{40}$$

As we can see from the above set of equations, two equations for  $\xi_{mz}$  and  $\xi_{mz}^*$  are algebraic in nature so we can use the third and the fifth equations to substitute  $\xi_{mz}$  and  $\xi_{mz}^*$  in other equations. Therefore, we have only three coupled fields equations for  $a_m$ ,  $\xi_{mt}$  and  $\xi_{mt}^*$  to be solved. Similarly we have another set of equations for  $a_p$ ,  $\xi_{pt}$ ,  $\xi_{pt}^*$ ,  $\xi_{pz}$  and  $\xi_{pz}^*$  components. These equations can be obtained simply by replacing  $m \rightarrow p$  and  $Q_1 \rightarrow -Q_1$  in the above.

Now we will solve the above set of equations numerically by integrating them from the horizon to the boundary of the bulk AdS black hole spacetime. There exists a well defined procedure to calculate the retarded Greens function of the dual boundary field theory [51, 53]. At the horizon, the ingoing wave boundary condition of all the fields are

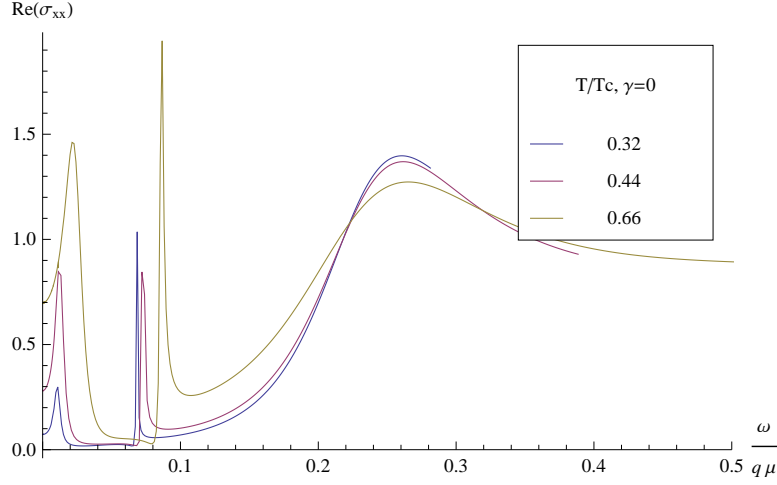


FIG. 6: The real part of the normal conductivity  $Re[\sigma_{xx}]$  vs.  $T/T_c$  for holographic d-wave superconductors ( $\gamma = 0$ ) with  $m^2 = 4$ . There are  $\delta(\omega)$  type supercurrent contributions in these curves that cannot be seen clearly. The hall conductivity  $\sigma_{xy}$  vanishes.

imposed to ensure causality:

$$\begin{aligned}
a_m &= f^{-i\frac{\omega}{3}} a_0 + \dots \\
\xi_{mt} &= f^{-i\frac{\omega}{3}} \xi_{mt0} + \dots \quad ; \quad \xi_{mt}^* = f^{-i\frac{\omega}{3}} \xi_{mt0}^* + \dots \\
\xi_{mz} &= f^{-i\frac{\omega}{3}-1} \xi_{mz0} + \dots \quad ; \quad \xi_{mz}^* = f^{-i\frac{\omega}{3}-1} \xi_{mz0}^* + \dots
\end{aligned} \tag{41}$$

Where  $a_0$ ,  $\xi_{mt0}$  and  $\xi_{mt0}^*$  are some arbitrary complex constants yet to be determined. Since the system in Eq. 40 is linear and homogeneous, we can set  $a_0 = 1$  because this only affects the overall normalization of  $\xi_{mt0}$  and  $\xi_{mt0}^*$ . It has no effect on conductivity. So by demanding  $\xi_{mt}$  and  $\xi_{mt}^*$  to be normalizable near the boundary as was done in Ref. [36],  $\xi_{mt0}$  and  $\xi_{mt0}^*$  are uniquely fixed. Therefore we can solve for the conductivity. This can all be done without the shooting method. Because the system is linear and homogeneous, the problem is reduced to taking the correct linear combination of three independent solutions to eliminate the non-normalizable terms of  $\xi_{mt}$  and  $\xi_{mt}^*$ . Once this is done, we can use the standard AdS/CFT technique to extract the retarded Greens function from the solution of the electromagnetic perturbation  $a_m$ .

Near the boundary,  $a_m$  has the asymptotic behavior

$$a_m(z) = a_m^0 + a_m^1 z + \dots, \tag{42}$$

where  $a_m^0$  corresponds to an applied electric field in the boundary theory and  $a_m^1$  corre-

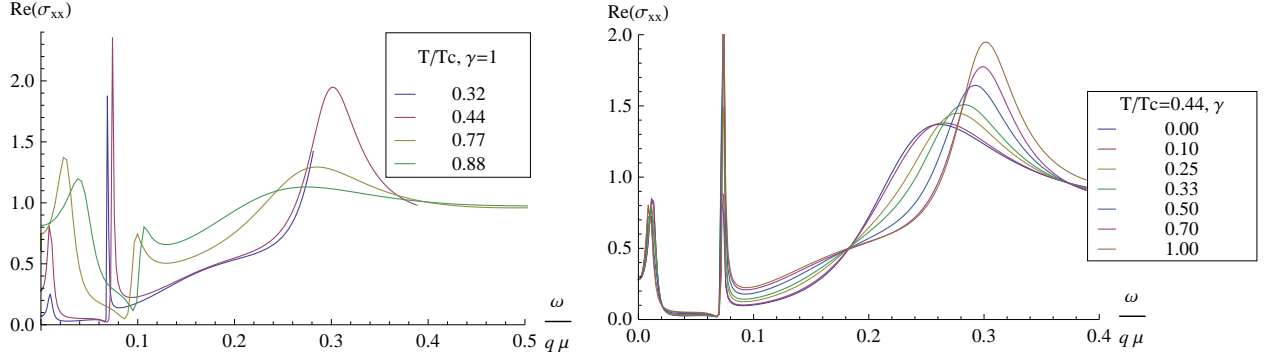


FIG. 7:  $Re[\sigma_{xx}]$  of holographic  $d + id$  superconductors ( $m^2 = 4$ ) for  $\gamma = 1$  with various  $T$  (left panel) and for  $T/T_c = 0.44$  with various  $\gamma$  (right panel). There are  $\delta(\omega)$  type supercurrent contributions in these curves that cannot be seen clearly.

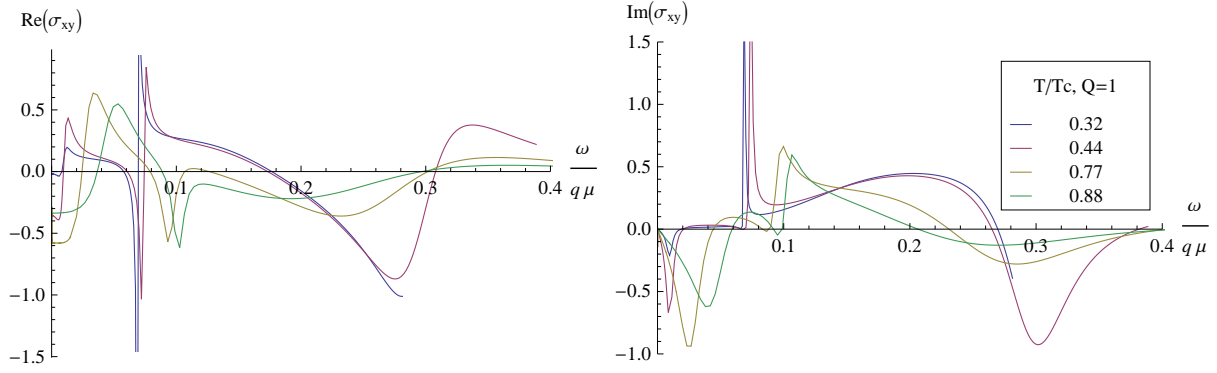


FIG. 8: The real and imaginary parts of the Hall conductivity,  $Re[\sigma_{xy}]$  and  $Im[\sigma_{xy}]$ , for  $d + id$  superconductors with  $m^2 = 4$ ,  $\gamma = 1$  and various  $T$ .

sponds to the induced current. The standard definition of the Greens function [51, 53] is

$$\sigma_+ = \sigma_{xx} + i\sigma_{xy} = \lim_{z \rightarrow 0} \frac{-ia_m^1}{\omega a_m^0}. \quad (43)$$

Then the standard conductivity tensor component would simply be

$$\sigma_{xx} = \frac{1}{2}(\sigma_+ + \sigma_-), \quad \sigma_{xy} = \frac{-i}{2}(\sigma_+ - \sigma_-). \quad (44)$$

In the d-wave case with  $\varphi_{xy} = \gamma = 0$ ,  $\sigma_{xy} = 0$ , and we have reproduced the  $\sigma_{xx}$  result of Refs.[35, 36] in Fig.6. For  $\gamma \neq 0$ , we have only shown the  $0 \leq \gamma \leq 1$  result in Figs.7-9 because  $\gamma \rightarrow 1/\gamma$  under a  $\pi/2$  rotation and then

$$\sigma_{ij}(\gamma) = \sigma_{ij}(1/\gamma). \quad (45)$$

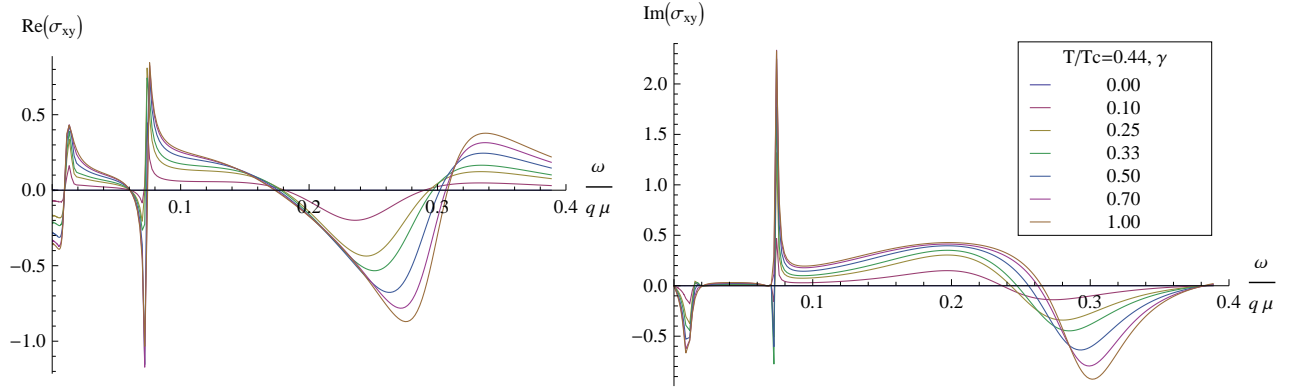


FIG. 9:  $Re[\sigma_{xy}]$  and  $Im[\sigma_{xy}]$  for various  $\gamma$  at  $m^2 = 4$  and  $T/T_c = 0.44$ .  $T_c$  is independent of  $\gamma$ .  $\sigma_{xy} = 0$  when  $\gamma = 0$ .

In each case, the real part of  $\sigma_{xx}$ ,  $Re(\sigma_{xx})$ , has a delta function type contribution  $\delta(\omega)$  from the super current if the system is in the superconducting phase. Also,  $Re(\sigma_{xx})$  is always non-negative. Below some frequency  $\omega_s$ ,  $Re(\sigma_{xx})$  could have several peaks, suggesting there are several spin one resonance states with masses set by the scale  $q\mu$ . Above  $\omega_s$ ,  $Re(\sigma_{xx})$  reaches its asymptotic value which is one. Numerically,  $\omega_s \sim q\mu$ . Unlike the s-wave SC's, the fermi surfaces are not gapped everywhere for the d-wave or  $d + id$  SC's. These conducting electrons can respond to an electric field of any frequency. Thus, there should be no gap in the conductivity. Our result also has this feature.

For  $Re(\sigma_{xy})$ , there is no  $\delta(\omega)$  contribution and it does not have a definite sign. At the peaks of  $Re(\sigma_{xx})$ ,  $Re(\sigma_{xy})$  vanishes (see Figs. 7 and 8). (But  $Re(\sigma_{xy})$  vanishes does not imply a peak at  $Re(\sigma_{xx})$ .) This is because suppose one of the diagonalized conductivity  $\sigma_+$  is much bigger than the other in size, then according to Eq. (44),  $Re(\sigma_{xy})$  is nothing but  $Im(\sigma_{xx})$ . So it relates to  $Re(\sigma_{xx})$  as described above.

In Figs. 8 and 9, the  $\sigma_{xy}$  for various  $T$  and  $\gamma$  is shown. It is interesting that within a small frequency window, the Hall conductivity  $Re(\sigma_{xy})$  could be very sensitive to the external electric field frequency with the Hall conductivity changes dramatically from a large negative value to a large positive value. Thus, it could be a good electromagnetic wave frequency sensor. About the  $\gamma$  dependence, there is a nearly universal  $Re(\sigma_{xx})$  in Fig. 7 when  $\omega/q\mu \simeq 0.18$ . Also, the shape of  $\sigma_{xy}$  in Fig. 9 is insensitive to  $\gamma$ . Those are curious features of this type of superconductors.

## VII. CONCLUSION

We studied a holographic model of  $d+id$  superconductors based on the action proposed by Benini, Herzog, and Yarom [35]. The model contains a charged spin two field in an AdS black hole spacetime. Working in the probe limit, the normalizable solution of the spin two field in the bulk gives rise to a  $d + id$  superconducting order parameter at the boundary of the AdS.

We have calculated the fermion spectral function in this superconducting background and confirmed the existence of fermi arcs for non-vanishing Majorana couplings. Depending on the relative strength  $\gamma$  of the  $d$  and  $id$  condensations, we found that the position and the size of the fermi arcs are changing. Specifically when we take  $\gamma = 1$ , the spectral functions become isotropic and are s-wave like. We also studied fermion mass effect. By changing the fermion mass, we saw the fermi momentum is changing. We have also calculated the conductivity for these holographic  $d+id$  superconductors where time reversal symmetry has been broken spontaneously. A non-vanishing Hall conductivity has been obtained even without an external magnetic field.

As we know in a real high temperature superconductor, the fermi arc has been observed in a pseudo gap phase before the superconducting phase transition. So far in the existing holographic models of d-wave superconductor the fermi arc appears in the superconducting phase. This is an interesting open problem to construct such a holographic model.

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- [1] P. A. Lee, N. Nagaosa, and X.-G. Wen, Rev. Mod. Phys. 78, 17 (2006), and references therein.
  - [2] C. C. Tsuei and J. R. Kirtley, Rev. Mod. Phys. 72, 969 (2000).
  - [3] Anderson, P. W., 1987, Science 235, 1196 (1987); Anderson, P. W. , The Theory of Superconductivity in the High Tc Cuprates, Princeton University Press, Princeton 1997.

- [4] Baskaran, G., and P. W. Anderson, Phys. Rev. B **37**, 580 (1988); Ioffe, L. B., and A. Larkin, Phys. Rev. B **39**, 8988 (1989); Nagaosa, N., and P. A. Lee, Phys. Rev. Lett. **64**, 2450 (1990); Wen, X.-G., and P. A. Lee, Phys. Rev. Lett. **76**, 503 (1996).
- [5] J. M. Maldacena, Adv. Theor. Math. Phys. **2**, 231 (1998) [Int. J. Theor. Phys. **38**, 1113 (1999)].
- [6] S. S. Gubser, I. R. Klebanov and A. M. Polyakov, Phys. Lett. B **428**, 105 (1998)
- [7] E. Witten, Adv. Theor. Math. Phys. **2**, 253 (1998)
- [8] G. Policastro, D. T. Son and A. O. Starinets, Phys. Rev. Lett. **87**, 081601 (2001)
- [9] C. P. Herzog, A. Karch, P. Kovtun, C. Kozcaz and L. G. Yaffe, JHEP **0607**, 013 (2006)
- [10] H. Liu, K. Rajagopal and U. A. Wiedemann, Phys. Rev. Lett. **97**, 182301 (2006)
- [11] S. S. Gubser, Phys. Rev. D **74**, 126005 (2006)
- [12] C. P. Herzog, P. Kovtun, S. Sachdev and D. T. Son, Phys. Rev. D **75**, 085020 (2007).
- [13] S. A. Hartnoll *et al.*, Phys. Rev. B **76**, 144502 (2007).
- [14] S. A. Hartnoll and C. P. Herzog, Phys. Rev. D **76**, 106012 (2007).
- [15] S. A. Hartnoll and C. P. Herzog, Phys. Rev. D **77**, 106009 (2008).
- [16] S. S. Gubser, Class. Quant. Grav. **22**, 5121 (2005).
- [17] S. S. Gubser, Phys. Rev. D **78**, 065034 (2008).
- [18] S. A. Hartnoll, C. P. Herzog and G. T. Horowitz, Phys. Rev. Lett. **101**, 031601 (2008).
- [19] S. A. Hartnoll, C. P. Herzog and G. T. Horowitz, JHEP **0812**, 015 (2008) [arXiv:0810.1563 [hep-th]].
- [20] G. T. Horowitz and M. M. Roberts, JHEP **0911**, 015 (2009) [arXiv:0908.3677 [hep-th]]; R. A. Konoplya and A. Zhidenko, Phys. Lett. B **686**, 199 (2010) [arXiv:0909.2138 [hep-th]]
- [21] S. S. Gubser, Phys. Rev. Lett. **101**, 191601 (2008) [arXiv:0803.3483 [hep-th]].
- [22] E. Nakano and W. Y. Wen, Phys. Rev. D **78**, 046004 (2008) [arXiv:0804.3180 [hep-th]].
- [23] T. Albash and C. V. Johnson, JHEP **0809**, 121 (2008) [arXiv:0804.3466 [hep-th]].
- [24] C. Y. Huang, F. L. Lin and D. Maity, arXiv:1102.0977 [hep-th]
- [25] J. W. Chen, Y. J. Kao and W. Y. Wen, arXiv:0911.2821 [hep-th].
- [26] T. Faulkner *et al.*, arXiv:0911.3402 [hep-th].
- [27] S. S. Gubser, F. D. Rocha and P. Talavera, arXiv:0911.3632 [hep-th].
- [28] S. S. Gubser and S. S. Pufu, JHEP **0811**, 033 (2008) [arXiv:0805.2960 [hep-th]].
- [29] M. M. Roberts and S. A. Hartnoll, JHEP **0808**, 035 (2008) [arXiv:0805.3898 [hep-th]].

- [30] M. Ammon *et al.*, Phys. Lett. B **686**, 192 (2010) [arXiv:0912.3515 [hep-th]]
- [31] P. Basu *et al.*, arXiv:0911.4999 [hep-th].
- [32] S. S. Gubser *et al.*, JHEP **1011**, 085 (2010) [arXiv:1002.4416 [hep-th]]; M. Ammon *et al.*, JHEP **1005**, 053 (2010) [arXiv:1003.1134 [hep-th]]; D. Vegh, arXiv:1007.0246[hep-th].
- [33] J. -W. Chen, Y. -J. Kao, D. Maity, W. -Y. Wen, C. -P. Yeh, Phys. Rev. **D81**, 106008 (2010). [arXiv:1003.2991 [hep-th]].
- [34] C. P. Herzog Phys. Rev. **D81**, 126009 (2010) [arXiv:1003.3278 [hep-th]]
- [35] F. Benini, C. P. Herzog, A. Yarom, [arXiv:1006.0731 [hep-th]].
- [36] F. Benini, C. P. Herzog, R. Rahman, A. Yarom, JHEP **1011**, 137 (2010). [arXiv:1007.1981 [hep-th]].
- [37] H. -B. Zeng, Z. -Y. Fan, H. -S. Zong, Phys. Rev. **D82**, 126014 (2010). [arXiv:1006.5483 [hep-th]].
- [38] H. -B. Zeng, Z. -Y. Fan, H. -S. Zong, Phys. Rev. **D82**, 126008 (2010). [arXiv:1007.4151 [hep-th]].
- [39] Q. Pan, B. Wang, [arXiv:1101.0222 [hep-th]].
- [40] M. Ammon *et al.*, Phys. Lett. B **680**, 516 (2009) [arXiv:0810.2316 [hep-th]]; M. Ammon *et al.* JHEP **0910**, 067 (2009) [arXiv:0903.1864 [hep-th]]; P. Basu *et al.* JHEP **0911**, 070 (2009) [arXiv:0810.3970 [hep-th]]; F. Denef and S. A. Hartnoll, Phys. Rev. D **79**, 126008 (2009). [arXiv:0901.1160 [hep-th]]; S. J. Rey, Prog. Theor. Phys. Suppl. **177**, 128 (2009) [arXiv:0911.5295 [hep-th]]; K. Peeters, J. Powell and M. Zamaklar, JHEP **0909**, 101 (2009). [arXiv:0907.1508 [hep-th]]; S. S. Gubser *et al.*, Phys. Lett. B **683**, 201 (2010) [arXiv:0908.0011 [hep-th]]; J. P. Gauntlett, J. Sonner and T. Wiseman, Phys. Rev. Lett. **103**, 151601 (2009) [arXiv:0907.3796 [hep-th]]; J. P. Gauntlett, J. Sonner and T. Wiseman, JHEP **1002**, 060 (2010) [arXiv:0912.0512 [hep-th]]; S. S. Gubser *et al.*, Phys. Rev. Lett. **103**, 141601 (2009) [arXiv:0907.3510 [hep-th]];
- [41] M. Covington, M. Aprili, E. Paraoanu, L. H. Greene, F. Xu, J. Zhu and C. A. Mirkin, “Observation of surface-induced broken time-reversal symmetry in YBa<sub>2</sub>Cu<sub>3</sub>O<sub>7</sub> tunnel junctions,” Phys. Rev. Lett. **79** (1997) 277.
- [42] R. Krupke and G. Deutscher, “Anisotropic magnetic field dependence of the zero-bias anomaly on in-plane oriented [100] Y<sub>1</sub>Ba<sub>2</sub>Cu<sub>3</sub>O<sub>7-x</sub>/In tunnel junctions,” Phys. Rev. Lett. **83** (1999) 4634.

- [43] R. Carmi, E. Polturak, G. Koren and A. Auerbach, “Spontaneous macroscopic magnetization at the superconducting transition temperature of  $\text{YBa}_2\text{Cu}_3\text{O}_{7-\delta}$ ,” *Nature* **404** (2000) 853.
- [44] G. Deutscher, Y. Dagan, A. Kohen and R. Krupke, “Field induced and spontaneous sub-gap in [110] and [100] oriented YBCO films: indication for a  $d_{x^2-y^2} + id_{xy}$  order parameter,” *Physica C* **341-348** (2000) 1629.
- [45] Y. Dagan and G. Deutscher, “On the origin of time reversal symmetry breaking in  $\text{Y}_{1-y}\text{Ca}_y\text{Ba}_2\text{Cu}_3\text{O}_{7-x}$ ,” *Europhys. Lett.* **57** (2002) 444.
- [46] R. B. Laughlin, “Magnetic induction of  $d_{x^2-y^2} + id_{xy}$  order in high- $T_c$  superconductors,” *Phys. Rev. Lett.* **80** (1998) 5188.
- [47] J. Goryo and K. Ishikawa, “Observation of induced Chern-Simons term in P- and T- violating superconductors,” *Phys. Lett.* **A260** (1999) 294.
- [48] I. L. Buchbinder, D. M. Gitman, and V. D. Pershin, *Phys. Lett.* **B492**, 161 (2000) [hep-th/0006144]; P. Federbush, *Il Nuovo Cimento* **19** 572 (1961); G. Velo, *Nucl. Phys.* **B43**, 389 (1972); M. Porrati and R. Rahman, arXiv:0809.2807 [hep-th]; M. Porrati and R. Rahman, *Nucl. Phys.* **B814**, 370 (2009) [arXiv:0812.4254 [hep-th]]; I. L. Buchbinder, D. M. Gitman, V. A. Krykhtin, and V. D. Pershin, *Nucl. Phys.* **B584**, 615 (2000) [hep-th/9910188]; M. Kobayashi and A. Shamaly, *Prog. Theor. Phys.* **61** 656 (1979); G. Velo and D. Zwanziger, *Phys. Rev.* **188**, 2218 (1969); Y. M. Zinoviev, *Nucl. Phys.* **B821**, 431 (2009) [0901.3462 [hep-th]].
- [49] N. T. Evans, *J. Math. Phys.* **8**, 170 (1967);  
G. Mack, *Commun. Math. Phys.* **55**, 1 (1977); S. Minwalla, *Adv. Theor. Math. Phys.* **2**, 781 (1998) [arXiv:hep-th/9712074].
- [50] H. Liu, J. McGreevy and D. Vegh, arXiv:0903.2477 [hep-th].
- [51] N. Iqbal and H. Liu, *Fortsch. Phys.* **57**, 367 (2009) [arXiv:0903.2596 [hep-th]].
- [52] M. Cubrovic, J. Zaanen and K. Schalm, *Science*, **325**, 439 (2009) [arXiv:0904.1993[hep-th]].
- [53] D. T. Son and A. O. Starinets, *JHEP* **0209**, 042 (2002) [arXiv:hep-th/0205051].
- [54] M. R. Norman et al., *Nature* **392**, 157 (1998).
- [55] A. Kanigel et al., *Nature Physics* **2**, 447 (2006).
- [56] A. Kanigel et al., *Phys. Rev. Lett.* **99**, 157001 (2007).